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Analysis of Friction Torque in Simple and Preloaded Spur Gear Trains

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ABSTRACT

The first part of this study develops the relation between the required drive torque and the load torque in a lightly loaded, single gear mesh, considering Coulomb friction between the sliding tooth surfaces. Bearing and windage losses are not considered. All of the load is assumed to be carried on a single pair of teeth, since for lightly loaded gears elastic deflections will be smaller than tooth-to-tooth errors, so that a single pair of teeth may carry the load regardless of the theoretical contact ratio. The use of "efficiency" formulas given in the literature usually gives low values for starting torque requirements.

The second part of the study extends the equations to a four-square gear train as representative of preloaded split gear trains, where friction losses are relatively greater than in simple trains. It is demonstrated that the maximum possible friction loss for a given four-square gear train would occur when and if the two gear meshes simultaneously begin approach action. It is also demonstrated that the friction loss for a gear train using spring-loaded split gears cannot exceed values calculated on the assumption that one mesh begins approach action while the other mesh terminates recess action.

PROBLEM STATUS

This is an interim report on one phase of the problem; work on this and other phases is continuing.

AUTHORIZATION

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ANALYSIS OF FRICTION TORQUE IN SIMPLE AND PRELOADED SPUR GEAR TRAINS

INTRODUCTION

The well-known analyses of the efficiency of a gear mesh published by Buckingham (1), Merritt (2), Shipley (3-5), and Tso (6) are primarily concerned with power losses. In this report, attention will be directed toward gear trains for data transmission and other uses where loads are light, and power loss, per se, is of minor importance. Designers of gear trains for data transmission, instruments, small servomechanisms, etc., are concerned with starting torque, since this directly affects spring-loading or bias requirements, and may often be the major load on a mechanical system. The use of standard formulas for friction power losses usually gives overoptimistic estimates of the starting torque requirements for gear trains.

The first part of this report will develop the friction torque relations in a single, lightly loaded gear mesh, with an effort to promote a clear understanding of the physical relationships. The second part extends the results to preloaded gear trains. It will be seen that torque required to overcome gear friction varies as gears rotate in mesh. The worst possible cases for friction will be emphasized throughout this paper, since design will usually have to provide for starting in just such a position. The symbols used follow Buckingham's notation so far as convenient.

FRICTION TORQUE IN A SINGLE GEAR MESH AS FOUND IN A SIMPLE GEAR TRAIN

Approach Action

Figure 1 shows a single gear mesh with the upper gear turning clockwise to drive a follower. The subscript 1 is used on the symbols referring to the driver, and the subscript 2 on the symbols referring to the follower. The subscript *a* refers to approach action, with the point of contact approaching *P* along *VZ*. The symbols are as follows:

R_{b1} = radius of base circle of driver (in.)

R_{b2} = radius of base circle of follower (in.)

VZ = the path of contact, of the gear mesh, with *v* the point of first contact and *Z* the point of last contact

UX = a common tangent between the base circles

ϕ = the pressure angle of the basic rack

R_{O1} = radius to tip of tooth of driver (in.)

R_{O2} = radius to tip of follower (in.)

f = coefficient of friction

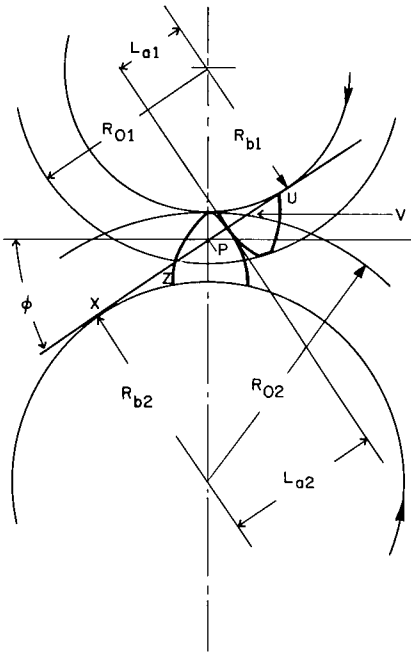


Fig. 1 - Single gear mesh during approach action (point of contact approaching P along VZ)

tion, then, is that a single tooth may carry the entire load on the gear mesh regardless of the theoretical contact ratio.

With the teeth meshing in the approach position shown, the normal force W_n opposes the rotation of the driver, while the frictional force fW_n exerts a moment in the direction of rotation. That is,

$$T_{a1} = W_n R_{b1} - fW_n L_{a1} = W_n (R_{b1} - fL_{a1}) \quad (1)$$

A free-body diagram of the driver is shown in Fig. 2.

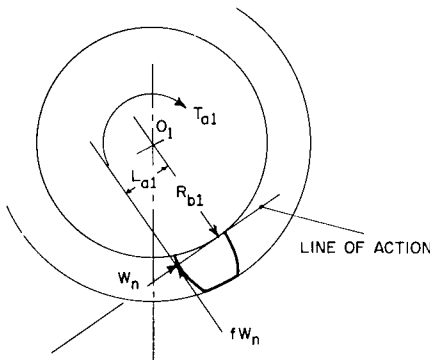


Fig. 2 - Free-body diagram of the driver gear during approach action

W_n = normal tooth load (lb)

L_{a1} = lever arm of friction force acting on driver during approach (in.)

L_{a2} = lever arm of friction force acting on follower during approach (in.)

T_{a1} = Torque exerted by driver during approach (in.-lb)

T_{a2} = Torque exerted on follower during approach (in.-lb)

m = gear ratio = N_2/N_1 .

The assumptions are as follows:

1. The teeth are involute form.
2. The friction force acts to oppose relative sliding between gear surfaces.
3. The magnitude of the friction force is proportional to the normal tooth force W_n and is independent of the sliding velocity except as noted.
4. The mesh is "lightly loaded." This means that elastic deflections will be smaller than the tooth-to-tooth errors. A corollary of this assumption, then, is that a single tooth may carry the entire load on the gear mesh regardless of the theoretical contact ratio.

Similarly, the normal force and the friction force on the follower tooth apply opposing torques during the approach action of the tooth. Equating these torques to the external load torque on the follower, we obtain

$$T_{a2} = W_n R_{b2} - fW_n L_{a2} = W_n (R_{b2} - fL_{a2}) \quad (2)$$

Then, from Eqs. (1) and (2), we obtain

$$\frac{T_{a1}}{T_{a2}} = \frac{R_{b1} - fL_{a1}}{R_{b2} - fL_{a2}} \quad (3)$$

Maximum friction torque is our main concern. For a given gear ratio, this means maximum ratio

of driver torque to follower torque. Since the coefficient f is being treated as a constant (assumption 3), the right-hand side of Eq. (3) will be maximum when L_{a1} is a minimum and L_{a2} a maximum. These conditions occur simultaneously at the beginning of the approach motion, at point V in Fig. 1.

By inspection of Fig. 1

$$L_{a2 \max} = XV = \sqrt{R_{O2}^2 - R_{b2}^2} \quad (4)$$

$$L_{a1 \min} = UV = UX - XV = (R_1 + R_2) \sin \phi - \sqrt{R_{O2}^2 - R_{b2}^2} \quad (5)$$

The maximum torque ratio may be found by using Eqs. (4) and (5) in Eq. (3).

If torque efficiency is defined as the ratio of the actual output torque to that which would be obtained with zero friction, we have

$$\text{efficiency} = \frac{T_{a2}}{mT_{a1}} = \frac{R_{b2} - fL_{a2}}{m(R_{b1} - fL_{a1})} \quad (6)$$

Let T_{f1} and T_{f2} be the friction torque, that is, the torque loss referred to driver and to follower speed, respectively:

$$\begin{aligned} T_{f2} &= mT_{a1} - T_{a2} \\ T_{f1} &= T_{a1} - \frac{T_{a2}}{m} = \frac{T_{f2}}{m} \end{aligned} \quad (7)$$

Using Eq. (3), we obtain

$$T_{f2} = T_{a1} \left(m - \frac{R_{b2} - fL_{a2}}{R_{b1} - fL_{a1}} \right) \quad (8)$$

A more compact expression for T_{f2} can be obtained in terms of W_n by using Eqs. (1) and (2) in Eq. (7):

$$T_{f2} = fW_n (L_{a2} - mL_{a1})$$

Maximum friction torque is obtained by using Eqs. (4) and (5):

$$T_{f2 \max} = fW_n (m+1)S$$

where $S = \sqrt{R_{O2}^2 - R_{b2}^2} - R_2 \sin \phi = \text{length of path of approach } \overline{VP}$.

Recess Action

Equations (1) through (3) apply only to approach action. Recess action will now be considered. The gear mesh of Fig. 1 is shown rotated to a position of recess action in Fig. 3. A free-body diagram of the driver is shown in Fig. 4 (neglecting bearing reactions). Note that the direction of the friction force is reversed from its direction during approach. Again we equate the moments of the forces acting on the teeth to the external torque on the respective gears:

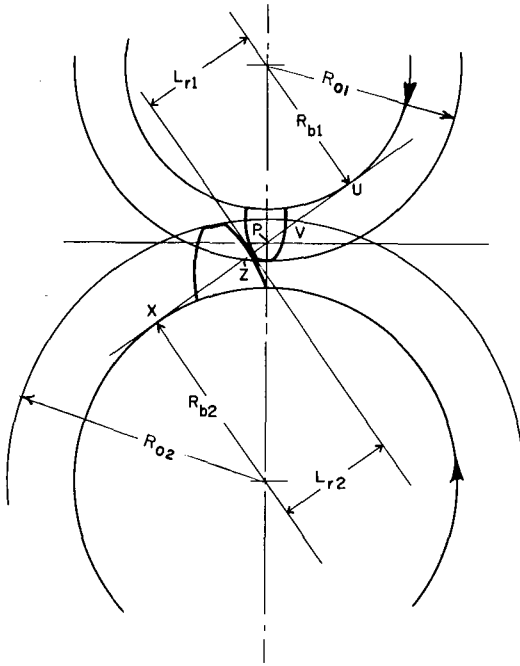
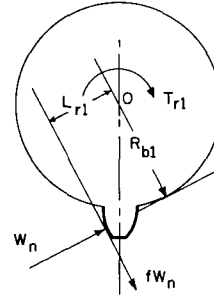


Fig. 3 - Single gear mesh during recess action (point of contact receding from P along VZ)

Fig. 4 - Free-body diagram of the driver gear during recess action



$$T_{r1} = W_n (R_{b1} + f L_{r1}) \quad (9)$$

$$T_{r2} = W_n (R_{b2} + f L_{r2}) \quad (10)$$

$$\frac{T_{r1}}{T_{r2}} = \frac{R_{b1} + f L_{r1}}{R_{b2} + f L_{r2}} \quad (11)$$

Maximum friction torque, which implies maximum ratio of T_{r1} to T_{r2} is still our main concern. Again considering f a constant, the maximum value of T_{r1}/T_{r2} will correspond to a maximum value for L_{r1} and a minimum for L_{r2} . It can be seen from Fig. 3 that these conditions occur simultaneously at point Z, the terminal point of recess action. At this point,

$$L_{r1_{\max}} = UZ = \sqrt{R_{o1}^2 - R_{b1}^2} \quad (12)$$

$$L_{r2_{\min}} = ZX = (R_1 + R_2) \sin \phi - \sqrt{R_{o1}^2 - R_{b1}^2} \quad (13)$$

Comparison of Friction Torques in Approach and Recess Action

For ordinary values of f , namely, from zero to 0.4, Eq. (3) gives a larger answer than Eq. (11). This means that, for a given value of f , the maximum torque required to overcome friction is somewhat greater in approach action than in recess action. If the value of f is actually greater during approach than during recess, as Buckingham (1) stated, the difference in torque is even greater than indicated by the comparison of these equations.

Inspection of Eq. (3) or Fig. 1 reveals that for a sufficiently high value of f , namely r_{b2}/L_{a2} , the follower gear would lock up, because the moment exerted by the normal tooth force would be opposed by an equal moment exerted by the friction force on the tooth. Needless to say, such high values of f would not be expected in practice.

Comment on the Relation of Other Published Analyses to This Work

The relations published for gear-mesh power efficiency by Buckingham (1), Merritt (2), and Shipley (4) differ slightly one from another in details. Merritt obtains an "instantaneous efficiency" by reasoning which neglects the difference between approach and recess action. This is essentially the same quantity called "torque efficiency" herein, given by Eq. (6). Assuming that the tooth load at the point of contact is constant, Merritt then integrates along the path of contact (vZ in Figs. 1 and 3) to obtain overall power efficiency. Instead of making Merritt's physical assumptions, one may obtain his equation for "instantaneous efficiency" from Eq. (6) of the present report by using the first two terms in a series expansion of the right-hand side and then neglecting the term containing the factor f^2 .

Buckingham gives an analysis which he credits to W. H. Clapp wherein the total work input to the follower is determined. The work efficiency or power efficiency is then the quotient of this quantity divided by the work input to the driver. Buckingham stresses the difference between approach and recess action, which Merritt considers "negligible." The relations given as Eqs. (3) and (11) are implicit in Buckingham's work. Shipley's analysis is similar to that of Merritt, whose work he lists among his references. Shipley's equations are given in the authoritative "Gear Handbook" (5). If any of the "efficiency formulas" mentioned be used for estimating maximum friction torque, the result obtained, for a given value of f , will be roughly 50 percent of the maximum values obtained using Eq. (8). This occurs because the calculation of work or power loss involves integration of triangular areas whose maximum altitude is proportional to the maximum friction torque.

All three authors assume, in effect, that each cycle of tooth engagement corresponds to the passage of a point of contact from end to end of the line of contact and that all of the load is carried on a single pair of teeth. Tso and Prowell (6) work out more general equations for friction power loss and efficiency which take into account the ideal distribution of load between teeth for any contact ratio. Their work does not allow, however, for the gearing errors which affect this distribution, namely, tooth spacing or pitch variation, profile error, and tooth thickness variation. For lightly loaded fine-pitch gears, it is easily shown that these errors are large compared with the tooth deflections that would serve to equalize loads. Thus, the assumption that all of the load is carried on a single pair of teeth is more realistic for the purposes of this report.

FRICION TORQUE IN A FOUR-SQUARE GEAR TRAIN REPRESENTATIVE OF BIASED, SPLIT GEAR TRAINS

The four-square gear train shown in Fig. 5 may be considered the basic split gear train. It consists of two pairs of meshing gears having the same size and ratio, secured to a pair of parallel shafts. If properly assembled, this gear train may be used to divide a transmitted load equally between two gear sets acting in parallel. On the other hand,

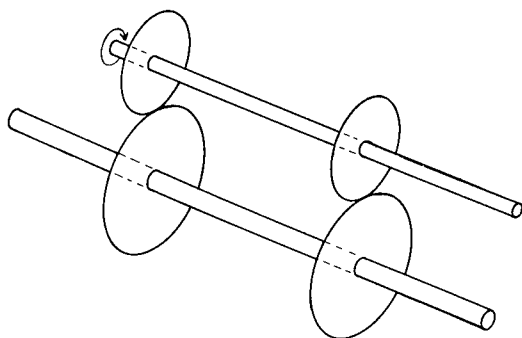


Fig. 5 - Four-square gear train

the gears may be adjusted to share the transmitted load unequally, or even so that the gears on a common shaft exert opposing torques. In the latter cases the four gears are under load even in the absence of any transmitted load. An adjustable coupling or other means may be used to establish this preloaded condition. Under preload, the four-square train is useful for drives where lost motion must be eliminated. The popular "antibacklash gearing" arrangement which uses a spring-loaded split gear meshed with a single pinion can be viewed as a special case of such a train, since the wide-faced pinion performs the functions of the two pinions shown in Fig. 5. The four-square train is also useful for

testing gears under heavy load because the drive torque input to the train need only be sufficient to overcome the friction.

Since friction is often one of the major components of the transmitted load in preloaded instrument gear trains, an analytical method is needed to predict friction torque, so that the proper preload may be selected and the various components correctly designed.

In the analysis of the gear train of Fig. 5, it is essential to observe the distinction between driver and follower. Suppose that a driving torque will be applied to one end, e.g., the left-hand end, of the upper shaft. This shaft is thus established as the driving shaft. If the gear train is biased so as to act as an effective antibacklash mesh, that is, with bias torque greater than any possible load torque, then only one of the gears on the

driver shaft functions as a driver. The other is driven by its mating gear on the "follower" shaft (lower shaft in Fig. 5). This is demonstrated by reference to Fig. 6, which is a set of free-body diagrams arranged as an exploded view of the gear train of Fig. 5. Bearing reactions are again neglected. The subscripts *A* and *B* will be applied to quantities related to the left-hand pair of gears and the right-hand pair, respectively. The torques shown in Fig. 6 are as follows:

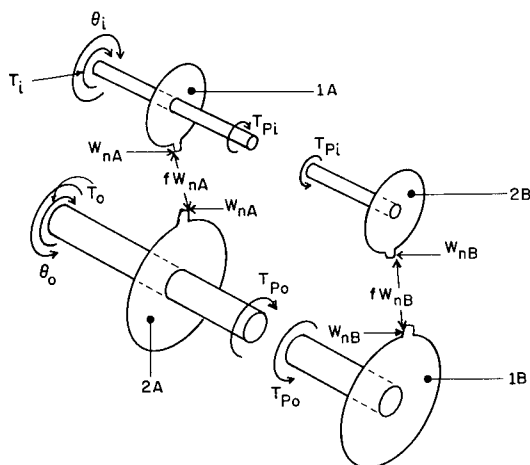


Fig. 6 - Free-body diagram of the four-square gear train

T_{pi} = preload torque in input, that is, driving shaft;

T_{po} = preload torque in output, that is, driven shaft;

T_i = input torque;

T_o = load torque (opposing motion).

Also shown in Fig. 6 are θ_i and θ_o indicating the assumed direction of rotation.

The preload process may be imagined to have been accomplished as follows:

1. The right-hand pair of gears are slipped out of mesh, by moving one of the gears axially.
2. The lower gear, $1B$, is held, while the upper gear, $2B$, is wound up in a clockwise direction.
3. The gears are slipped back into mesh.

For the clockwise rotation θ_i indicated in the figure, the left-hand upper gear is a driver, but the right-hand upper gear is a follower. As before, the subscripts 1 and 2 will indicate driver and follower, respectively.

It has been seen that the equations of torque equilibrium for a single mesh depend upon whether the mesh action is approach or recess type. For the four-square gear train we then have three cases: both meshes in approach action, one mesh in approach action and other mesh in recess action, and both meshes in recess action. The conditions under which these cases occur will be discussed later.

Both Meshes in Approach Action

First considering the case of both meshes in approach action, the equations of torque equilibrium are written for the right-hand plane, designated B, using the same reasoning used for Eqs. (1) and (2):

$$T_{pi} = W_{nB} R_{b2B} - f W_{nB} L_{a2B} = W_{nB} (R_{b2B} - f L_{a2B}) \quad (14)$$

$$T_{po} = W_{nB} (R_{b1B} - f L_{a1B}) . \quad (15)$$

Similarly, for plane A, the left-hand plane,

$$T_i + T_{pi} = W_{nA} (R_{b1A} - f L_{a1A}) \quad (16)$$

$$T_o + T_{po} = W_{nA} (R_{b2A} - f L_{a2A}) . \quad (17)$$

We will solve for T_i , the torque necessary to drive this gear train in the direction shown:

$$T_i = W_{nA} (R_{b1A} - f L_{a1A}) - W_{nB} (R_{b2B} - f L_{a2B}) . \quad (18)$$

But, by Eq. (17)

$$W_{nA} = \frac{T_o + T_{po}}{R_{b2A} - f L_{a2A}}$$

and by Eq. (15)

$$W_{nB} = \frac{T_{po}}{R_{b1B} - f L_{a1B}} .$$

Hence

$$T_i = T_o \left(\frac{R_{b1A} - f L_{o1A}}{R_{b2A} - f L_{a2A}} \right) + T_{po} \left[\frac{R_{b1A} - f L_{a1A}}{R_{b2A} - f L_{a2A}} - \frac{R_{b2B} - f L_{a2B}}{R_{b1B} - f L_{a1B}} \right] . \quad (19)$$

For the ordinary four-square gear train as shown in Fig. 6, assuming all gears cut with the same nominal pressure angle,

$$R_{b2B} = R_{b1A} \text{ and } R_{b1B} = R_{b2A}.$$

Then T_i may be expressed as

$$T_i = T_o P + T_{po} Q \quad (20)$$

where

$$P = \frac{R_{b1A} - fL_{a1A}}{R_{b2A} - fL_{a2A}} \quad (21)$$

$$Q = \frac{R_{b1A} - fL_{a1A}}{R_{b2A} - fL_{a2A}} - \frac{R_{b1A} - fL_{a2B}}{R_{b2A} - fL_{a1B}}. \quad (22)$$

This equation does not express the relation between the parameters in a way easy to visualize. In order to obtain an expression easier to interpret, each of the quotients indicated in Eq. (22) is expressed as a power series. Neglecting the third term and higher terms of the series, and collecting terms, a new expression is obtained:

$$Q \approx \frac{f}{R_{b2A}} (L_{a2B} - L_{a1A}) + \frac{f}{R_{b2A}^2} [R_{b1A} (L_{a2A} - L_{a1B}) + f(L_{a1B}L_{a2B} - L_{a1A}L_{a2A})] \quad (23)$$

$$Q \approx \frac{f}{R_{b2A}} \left[L_{a2B} - L_{a1A} + \frac{L_{a2A} - L_{a1B}}{m} + \frac{f(L_{a1B}L_{a2B} - L_{a1A}L_{a2A})}{R_{b2A}} \right]. \quad (24)$$

If the terms multiplied by the factor f^2 are disregarded, then, if the gears are timed so that approach action begins simultaneously on both meshes, Q_{\max} may be simplified to

$$Q_{\max} = \left(\frac{m+1}{m} \right) \frac{f}{R_{b2A}} [L_{a2A_{\max}} + L_{a2B_{\max}} - \overline{CD} \sin \phi] \quad (25)$$

where \overline{CD} is the center distance of the gears and $L_{a2A_{\max}}$ and $L_{a2B_{\max}}$ are given by Eqs. (4) and (5).

Q will be maximum, clearly, if L_{a2A} and L_{a2B} simultaneously reach a maximum. Whether this can occur will depend upon the relative timing of the gears in plane A and plane B.

One Mesh in Approach Action and the Other in Recess Action

For the next case, it will be assumed that the gears in plane A are in approach action while those in plane B are in recess action. The equations of torque equilibrium for plane B are then similar to Eqs. (9) and (10), bearing in mind that in plane B the gear on the lower shaft is the driver. On gear 2B we have

$$T_{pi} = W_{nB} (R_{b2B} + fL_{r2B}). \quad (26)$$

On gear 1B we have

$$T_{po} = W_{nB} (R_{b1B} + fL_{r1B}). \quad (27)$$

Using Eq. (16) for gear 1A we have

$$T_i = W_{nA}(R_{b1A} - fL_{a1A}) - W_{nB}(R_{b2B} + fL_{r2B}). \quad (28)$$

Using Eqs. (17) and (27) to eliminate W_{nA} and W_{nB} from Eq. (28) we obtain

$$T_i = T_o P + T_{po} Q' \quad (29)$$

where P is given by Eq. (21) and

$$Q' = \frac{R_{b1A} - fL_{a1A}}{R_{b2A} - fL_{a2A}} - \frac{R_{b1A} + fL_{r2B}}{R_{b2A} + fL_{r1B}}. \quad (30)$$

The approximate equation for Q' is obtained in the same way as Eq. (24):

$$Q' \approx \frac{f}{R_{b2A}} \left[-(L_{a1A} + L_{r2B}) + \frac{L_{a2A} + L_{r1B}}{m} + \frac{f(L_{r2B}L_{r1B} - L_{a1A}L_{a2A})}{R_{b2A}} \right]. \quad (31)$$

Both Meshes in Recess Action

Similar equations can be obtained for the case where both meshes are simultaneously in recess action. Since the driving torque must be greater during approach action, this case is less important and will accordingly not be developed here.

Maximum Driving Torque

Driving torque T_i is evidently maximum when and if P and Q or Q' simultaneously obtain their maximum values. The conditions for maximum of the term P were given in Eqs. (4) and (5). It can be seen that maximum Q or Q' occurs if the second term of the right-hand side of Eq. (22) or Eq. (30), respectively, is minimum when P is maximum.

Thus for the case of both meshes in approach action the driving torque T_i would have the maximum possible value if and when L_{a2A} and L_{a2B} simultaneously have maximum value. This situation occurs if both meshes simultaneously initiate their approach action. (Note that L_{a1A} and L_{a1B} will always be minimum when L_{a2A} and L_{a2B} are respectively at maximum, since $L_{a1} + L_{a2} = \overline{CD} \sin \phi$.) The appropriate values for the L quantities in Eqs. (21) and (22) are given by Eqs. (4) and (5).

For the case of mesh A in approach action and mesh B in recess action the driving torque T_i would have the maximum possible value if and when L_{a2A} and L_{r1B} simultaneously have maximum value. This situation occurs when mesh A begins approach action and, simultaneously, mesh B terminates recess action. (Note that L_{r2B} will automatically be minimum when L_{r1B} is maximum, since $L_{r1} + L_{r2} = \overline{CD} \sin \phi$.) The appropriate values for the L quantities in Eqs. (21) and (30) are given in Eqs. (4), (5), (12), and (13). It can be shown that this meshing condition will occur if the gear tooth proportions satisfy the equality

$$T_t + nP_c \frac{R_{O2A}}{R_{2A}} = \frac{2R_{O2A}}{R_{b2A}} \left(\sqrt{R_{O2A}^2 - R_{b2A}^2} - R_{2A} \sin \phi \right) \quad (32)$$

where

T_t = tooth thickness at the tip of both gear 2A and gear 1B,

P_c = circular pitch,

$$n = 0, 1, 2, \dots,$$

R_{2A} = nominal pitch diameter of gear 2A.

Only a few of the possible gear combinations exactly satisfy this criterion; a much larger proportion come fairly close. Thus the estimate of required driving torque obtained by using the maximum described will usually be conservative.

Application to Design Calculation

Given the output torque T_o for any four-square gear train, a conservative estimate of the required input torque is obtained by assuming that the train can pass through the maximum value for the case of both meshes in approach action. The input or drive torque can then be calculated by Eqs. (20), (21), and (22), using the values for the lever arms as given by Eqs. (4) and (5). The possibility of this occurrence depends upon the timing between the gears in plane A and those in plane B. If this timing is not specified in a split train, it would be necessary for a conservative design to assume that this maximum can occur.

If, however, the four-square gear train under consideration takes the specific form of a set of spring-loaded split gears, then, as noted earlier, pinion 2B is merely an extension of pinion 1A. For such an arrangement, the maximum possible drive torque is the maximum given for the case of mesh A in approach action and mesh B in recess action by using in Eq. (29) the values for the lever arms given in Eqs. (4), (5), (12), and (13). This maximum could not actually occur unless the design happened to satisfy Eq. (32). For nearly all design applications, however, the conservative answer given would have accuracy consistent with the other design parameters.

A designer might have a real need to minimize friction in a four-square gear train. The present study does not give a systematic solution to such a problem. The previous section, however, makes it evident that maximum starting friction conditions can be avoided, other things being equal, by avoiding designs which make it possible for the two meshes to simultaneously begin approach action or for one mesh to begin approach action while the other mesh terminates recess action (Eq. (32)). Tooth numbers and other parameters can be chosen so as to approach a minimum condition, using the relations given for guidance and for verification of choice.

A review of the extensive literature on friction coefficients is beyond the scope of this report. Note, however, that values of f used to obtain starting torque should be higher than those given in the literature for well-lubricated gearing. Lubrication will be boundary type, at best, since the steady bias torque has time to squeeze out the lubricant when the gear train is stationary. If better data are not available, a conservative value of $f = 0.2$ is suggested for free-running instrument gear trains with grease lubrication.

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13. ABSTRACT			
<p>The first part of this study develops the relation between the required drive torque and the load torque in a lightly loaded, single gear mesh, considering Coulomb friction between the sliding tooth surfaces. Bearing and windage losses are not considered. All of the load is assumed to be carried on a single pair of teeth, since for lightly loaded gears elastic deflections will be smaller than tooth-to-tooth errors, so that a single pair of teeth may carry the load regardless of the theoretical contact ratio. The use of "efficiency" formulas given in the literature usually gives low values for starting torque requirements.</p> <p>The second part of the study extends the equations to a four-square gear train as representative of preloaded split gear trains, where friction losses are relatively greater than in simple trains. It is demonstrated that the maximum possible friction loss for a given four-square gear train would occur when and if the two gear meshes simultaneously begin approach action. It is also demonstrated that the friction loss for a gear train using spring-loaded split gears cannot exceed values calculated on the assumption that one mesh begins approach action while the other mesh terminates recess action.</p>			

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Gears						
Analysis						
Friction						
Drives						
Torque						
Loading (mechanics)						
Moments						
Rotation						
Instrumentation						
Servomechanisms						
Data transmission systems						
Four-square gear train						

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